The inverse scattering transform and meromorphic solutions to the KdV equation with non-decaying initial data

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Abstract:

We are concerned with the Cauchy problem for KdV equation on the whole line with initial profiles which are decaying sufficiently fast on the right half line and arbitrary (with on decay assumption) on the left half line. We show that a reflection coefficient can be suitably defined in terms of the Titchmarsh-Weyl m-functions associated with the two half-line Schrodinger operators. In the short-range case our reflection coefficient agrees with the standard one. It is however well defined for much larger class of potentials as it is composed of Titchmarsh-Weyl m-functions which exist under almost no restriction on the potential. Moreover, since the m-function has a Herglotz property in the upper half plane, the reflection coefficient also has a certain analytic structure. Although the transmission coefficient need not exist but nevertheless the analyticity allows one to push the inverse scattering transform (IST) method far beyond the standard decay assumptions. The Marchenko integral equation of the inverse scattering theory can then be suitably regularized and the machinery of the IST smoothly works (including existence, uniqueness, well posedness, etc.). Our analysis of the Marchenko equation is based on certain subtle properties of the Titchmarsh-Weyl m-function. We show that the solution obtained this way represents a meromorphic function of the spatial variable. Among others, our approach yields some relevant results due to Cohen, Kappeler, Khruslov, Venakides, and others.